Soft Mathematical Morphology For Binary Images Filtering

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Abstract

The paper proposes soft morphological operators and algorithms to remove noise from binary images. The features of the algorithm are the presence of additional parameters, such as the threshold of filtering and number of iterations, and the ability to remove noise: islands, pin holes, mousebites and spurs in one pass. Application of these algorithms can bring less distortion during operation, even if the image objects are spaced one pixel apart. There are results of the experiments on the example of the PCB artwork.

1. Introduction

The basic operators of classical mathematical morphology – erosion and dilation [10, 19, 30] is expressed as

$$\mathrm{Dilate}(A,B) = \cup \{A + b : b \in B\}, \quad (1)$$

$$\mathrm{Erode}(A,B) = \cap \{A - b : b \in B\}, \quad (2)$$

where A – an original image, b – pixels of a structuring element B.

To remove noise from the image can be applied filters based on the open and close operations, which are a sequence of erosion and dilation

$$\mathrm{Open}(A,B) = \mathrm{Dilate}(\mathrm{Erode}(A,B),B)\,. \quad (3)$$

$$\mathrm{Close}(A,B) = \mathrm{Erode}(\mathrm{Dilate}(A,-B),-B). \quad (4)$$

In [11, 12] were offered soft morphological operators – soft erosion and dilation of the signal $f : \mathbb{Z}^m \to \mathbb{R}$ by structuring systems $[B_1, B_2, k]$ and filters based on them:

$$f \ominus [B_1,B_2,k]\,(x) = k\text{- }th\ smallest\ value\ of\ multiset$$
$$\{k \lozenge f(b_2) : b_2 \in B_2(x)\} \cup \{f(b_1) : b_1 \in (B_1(x))\}$$
$$\text{for all } x \in \mathbb{Z}^m \quad (5)$$

$$f \oplus [B_1,B_2,k]\,(x) = th\ biggest\ value\ of\ multiset$$
$$\{k \lozenge f(b_2) : b_2 \in B_2(x)\} \cup \{f(b_1) : b_1 \in (B_1(x))\}$$
$$\text{for all } x \in \mathbb{Z}^m. \quad (6)$$

The structuring system $[B1,\ B2,\ k]$ consists of three parameters: the finite sets B_1 and B_2, $B_2 \in B_1$, wherein $B_1, B_2 \in \mathbb{Z}^m$, positive integer k satisfies $1 \le k \le \mathrm{Card}(B_1)$. Set $B = B_1 \cup B_2$, $B_1 \cap B_2 = \varnothing$ is called structuring set, B_2 – a center, B_1 – an abroad, k – ordinal index of the center (parameter of repeatability).

In [18, 20] proposed following definition of operators of binary soft morphology:

$$A \ominus [B_1, B_2, k]\,(x) = \{x \in A \mid$$
$$(k \times \mathrm{Card}[A \cap (B_1)_x] + \mathrm{Card}[A \cap (B_2)_x] \ge$$
$$\ge k \times \mathrm{Card}[B_1] + \mathrm{Card}[B_2] - k + 1\}, \quad (7)$$

$$A \oplus [B_1, B_2, k](x) = \{\, x \in A \mid$$
$$\mid (k \times \mathrm{Card}[A \cap (B_1^S)_x] + \mathrm{Card}[A \cap (B_2^S)_x] \ge k, \quad (8)$$

where k – ordinal index, which determines how many times the kernel elements are recorded in the final result. If $k = 1$ or $B = B_1$ ($B_2 = \varnothing$), the operators of the soft morphology turn to standard mathematical morphology operators.

The operators of mathematical morphology based on the fuzzy sets, proposed in [21]. In this approach, fuzziness is determined by inscribe of the structuring element into the picture. To date, there is no single approach to determine the fuzzy erosion and dilation. In [2–6, 14, 15–17, 21, 25] are different definitions of basic operations of fuzzy morphology, characterized by the results of the work. For example, according to [21], the operators of fuzzy erosion and dilation by a fuzzy structuring element can be written as follows (in terms of membership functions):

$$\mu_{A \ominus B}(x) = \min_{y \in B}[\min[1,\ 1 + \mu_A(x+y) - \mu_B(y)]]$$
$$= \min[1,\ \min_{y \in B}[1 + \mu_A(x+y) - \mu_B(y)]], \quad (9)$$

$$\mu_{A \oplus B}(x) = \max_{y \in B}[\min[0,\ \mu_A(x-y) + \mu_B(y) - 1]]$$
$$= \max[0,\ \max_{y \in B}\ [\mu_A(x-y) + \mu_B(y) - 1]], \quad (10)$$

where $x, y \in \mathbb{Z}^2$ – spatial coordinates, μ_A and μ_B – membership functions of the image and the structuring element.

In [1, 8, 9] describes the approach, combining a soft and fuzzy morphology. Fuzzy soft erosion and dilation can be expressed as follows (the definition of fuzzy morphology was used according to [21]):

$$\mu_{A \ominus [B1,\,B2,\,k]}(x) = \min[1, \min_{y \in B1,\, z \in B2}{}^{(k)}(\{k \lozenge (\mu_A(x+y) - \mu_{B1}(y) + 1)\} \cup \{\mu_A(x+z) - \mu_{B2}(z) + 1\})], \quad (11)$$

$$\mu_{A \oplus [B1,\,B2,\,k]}(x) = \max[0, \max_{(x-y) \in B1,\, (x-z) \in B2}{}^{(k)}(\{k \lozenge (\mu_A(x-y) + \mu_{B1}(y) - 1)\} \cup \{\mu_A(x-z) + \mu_{B2}(z) - 1\})], \quad (12)$$

where $x, y, z \in \mathbb{Z}^2$– spatial coordinates, μ_A, μ_{B1} and μ_{B2} – membership function of image A, a core B_1 and soft boundary B_2 of structuring element. For a fuzzy structuring element is satisfied the conditions $B \in \mathbb{Z}^2 : B = B_1 \cup B_2$, $B_1 \cap B_2 = \varnothing$. If $k = 1$ the operators of the fuzzy soft morphology turn to standard mathematical morphology operators.

It should be noted that the publications in modern literature confirm the actuality of soft [13, 23, 24, 26] and fuzzy [7, 15, 22, 27] morphology imaging.

2. Image filtering algorithms based on the operators of soft morphology

In [28, 29] were proposed soft morphological operators, which combine elements of soft and fuzzy morphology:

1) likewise as the soft mathematical morphology, the proposed structuring element consists of a soft border, but the core of structuring element is the empty subset;

2) likewise as the fuzzy morphology, the principle of operation is based on the ability to fit the structuring element in the image, but the item is not considered as the fuzzy set.

The term "soft" is used to indicate, what different results can be obtained with the same structuring element and varying threshold of filtering.

The operators of soft morphology defined as follows:

$$SoftErode(A,B,t) = \left\{ \begin{matrix} n_s + t \le n_n \to a = 0 \\ n_s + t > n_n \to a = 1 \end{matrix}, a \in A \right\}, \quad (13)$$

$$SoftDilate(A,B,t) = \left\{ \begin{matrix} n_s + t \le n_n \to a = 1 \\ n_s + t > n_n \to a = 0 \end{matrix}, a \in A \right\}, \quad (14)$$

where $n_s = \sum_i (a_i \wedge b_i)$ – a number of non-zero pixels of the image in the mask of a structuring element, which coincided with the same pixels of the structuring element, $n_n = \sum_i (a_i \wedge b_i)$ – the number of not coincided pixels of image and the structuring element, t – the threshold of filtering, a – pixels of the original black and white image A, b – the pixels of a flat structuring element B. The term "a flat structuring element" indicates what it's pixels can be 0 or 1 only.

The initial data and result are the binary images. Operating parameters are a size and a shape of the structuring element, the value of the threshold.

To remove noise from images is used the operation of soft open and close, which can be written as follows:

$$\text{SotfOpen}(A, B, t) = \text{SoftDilate}(\text{SoftErode}(A, B, t), B, t), \quad (15)$$

$$\text{SotfClose}(A, B, t) = \text{SoftErode}(\text{SoftDilate}(A, B, t), B, t). \quad (16)$$

The initial data and result are the binary images. Operating parameters are a size and a shape of the structuring element, the value of the threshold and the number of filter passes.

The soft open and close are not idempotent, so can be re-used with the same structuring element many times, and it is proposed to use this property for the construction of soft morphological filters. Algorithm for image filtering based on the soft open and close is shown in Figure 1.

The filter parameters are the structuring element B (its size and shape), the threshold t and the number of filter iterations i.

It is possible to use soft sequential morphological filters, which are denoted *SOC*, *SCO*, *SOCO*, *SCOC* etc., where abbreviations *SO* and *SC* designated operation of soft open and close. These filters smooth out the jagged edges of elements similar to filters of classical mathematical morphology. And with the proposed soft morphology operators we can use filters $i_{so}SO$, $i_{sc}SC$, $i_{so}SOi_{sc}SC$, $i_{sc}SCi_{so}SO$ etc., where i_{so} and i_{sc} – the numbers of iterations of soft open and close, respectively. The number of filter passes is determined depending on the quality of the input image.

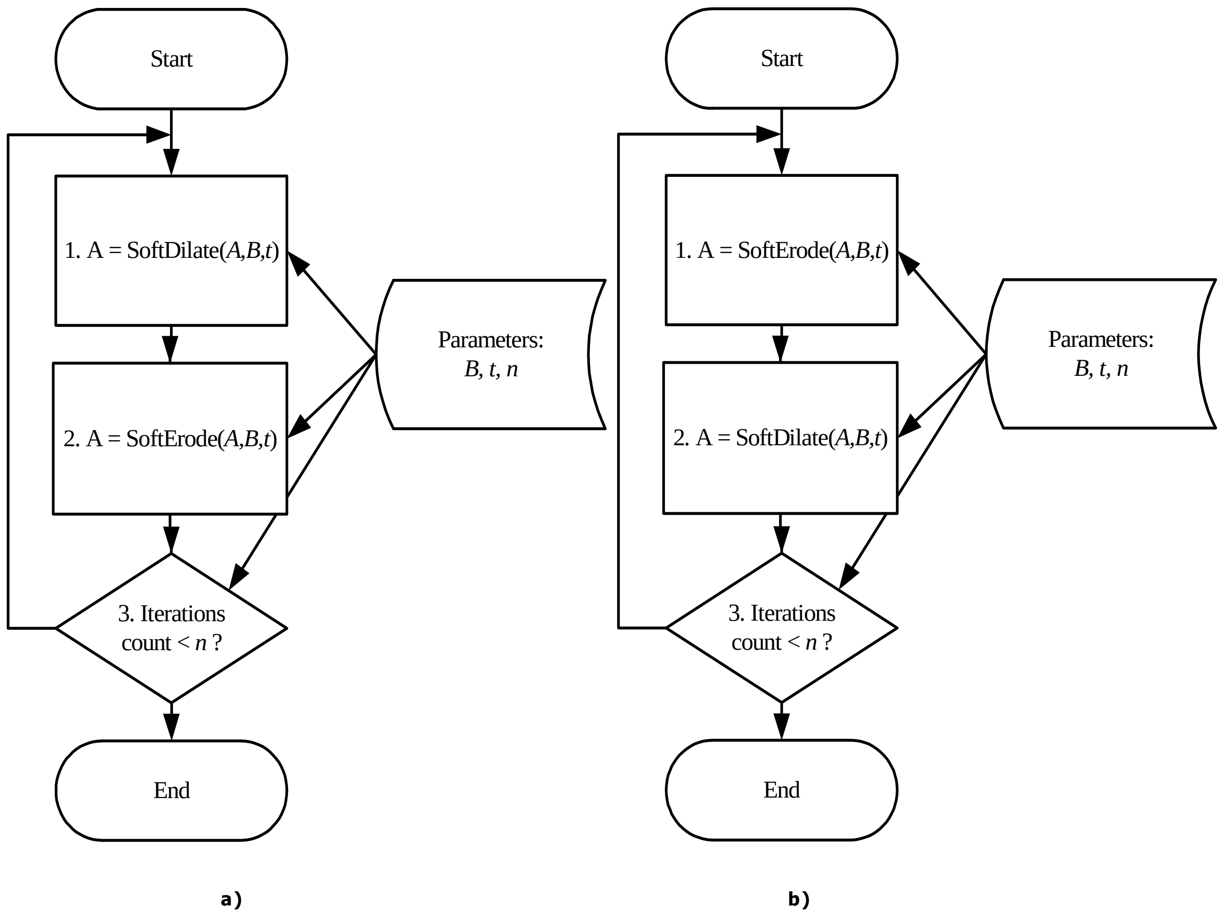

Fig. 1. Algorithms of soft morphological filtering operations: (a) SoftClose(*A*,*B*,*t*), (b) SoftOpen(*A*,*B*,*t*)

3. Experiments

On the example of the PSB artwork consider the advantages and characteristics of the soft morphological filter. An initial image is a fragment of the PSB artwork with the noise (Fig. 2, a). Image size is 1596 x 1007 pixels (67,57 mm x 42,63 mm, one image pixel corresponds to 42.3 microns). The result of the soft open operation by the square structuring element with 3 x 3 pixels size and with a threshold equal 0 is shown in Fig. 2, b. Results of classical operations of open and close on the same structuring element in Fig. 2, c and Fig. 2, d, respectively. Assessment of filtration quality (Tab. 1) was calculated as

$$Q = 1 - \mathrm{Card}(Test\ \mathrm{XOR}\ Etalon) / \mathrm{size}\ (Test), \quad (17)$$

where *Test* is the result of filtering, *Etalon* is a reference PSB artwork without noise. Evaluation is given in Tables 1 and 2.

Classical Open (*A, B*) operation removes islands from the image; smoothes spurs, increases mousebites, forms open circuits. A classic Close (*A, B*) operation removes pin holes from the image; smoothes mousebites, increases spurs, forms short circuits.

The high quality filtration needs to remove both islands and pin holes, to smooth both mousebites and spurs. Therefore, we can apply the sequential filter CloseOpen (*A, B*) (Fig. 2, d) or OpenClose (*A, B*) (Fig. 2, e). Assessment of filtering quality of these operations is shown in Tab. 2.

Classic sequential filter removes noise better than the filter, which uses only one Open (*A, B*) or Close (*A, B*) operation. Soft sequential filter removes noise better than classical sequential filter. However, filter based on only one SoftOpen(*A, B, t*)or SoftCloseOpen(*A, B, t*) operation removes both islands and pin holes, as well as smoothes both mousebites and spurs in one pass (Fig. 2, b). So it is expedient to choose simple soft morphology filter, if needed a high-speed operation. For example, when a slightly improvement of the quality of filtration (e.g., from 0.99286 to 0.99462, as is the case with operations SoftOpen(*A, B, t*) and SoftCloseOpen(*A, B, t*)) is less advantageous than the reduction of calculations in 2 times.

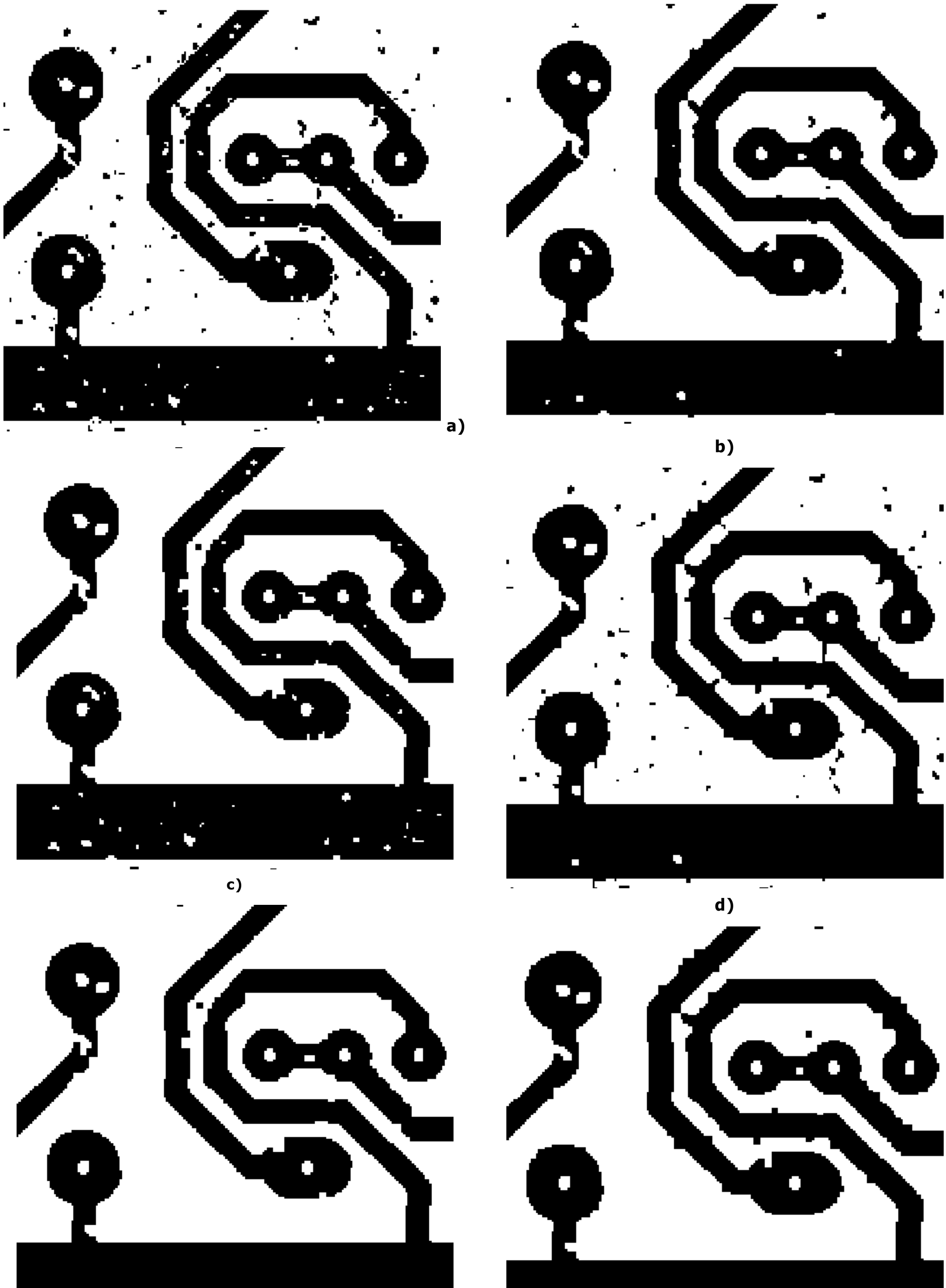

Fig. 2. Initial image (a), the result of the soft open operation by the square structuring element with 3 x 3 pixels size: (b) SoftOpen(*A,B*,0) - threshold is equal 0, (c) Open(*A,B*), (d) Close(*A,B*), (e) CloseOpen(*A,B*), (f) OpenClose(*A,B*)

Tab.1.

The quality of filtering using the structuring element with 3 x 3 pixels size

Filtering threshold t	Filtering quality			
	SoftOpen(A,B,t)		SoftClose(A,B,t)	
	B – square	B – diamond	B – square	B – diamond
0	0.99286	0.98481	0.99286	0.98876
2	0.99172	0.9901	0.99116	0.99135
4	0.99254	0.98795	0.99045	0.98795
6	0.99167	0.99135	0.98832	0.9901
8	0.99061	0.98876	0.98532	0.98481
Classical morphology	Open(A,B) 0.99061	Open(A,B) 0.98876	Close(A,B) 0.98532	Close(A,B) 0.98481

Tab.2.

The quality of sequential morphological filters by the structuring element with 3 x 3 pixels size

Filtering threshold t	Filtering quality			
	SoftOpenClose(A,B,t)		SoftCloseOpen(A,B,t)	
	B – square	B – diamond	B – square	B – diamond
0	0.99462	0.99086	0.99462	0.99067
2	0.99101	0.99352	0.99127	0.99338
4	0.99164	0.98804	0.99191	0.98804
6	0.99356	0.99338	0.99363	0.99352
8	0.99382	0.99067	0.99397	0.99086
Classical morphology	OpenClose(A,B) 0.99382	OpenClose(A,B) 0.99067	CloseOpen(A,B) 0.99397	CloseOpen(A,B) 0.99086

Fig. 3 shows the fragment of noisy PSB artwork with dedicated areas of open and short circuit.

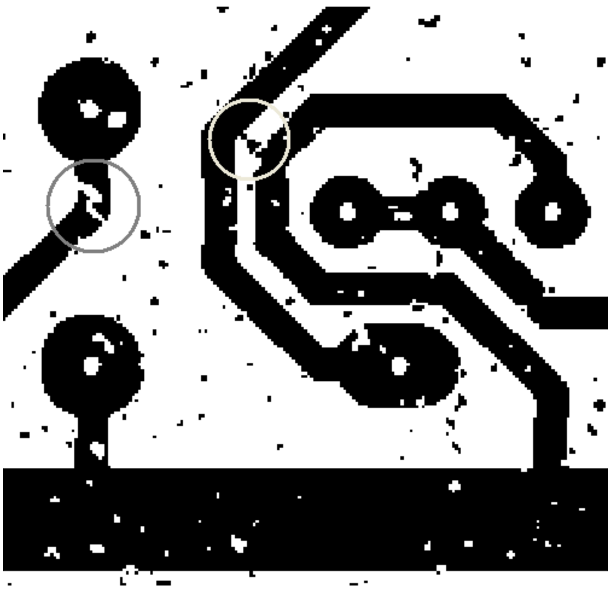

Fig. 3. A fragment of the mask with dedicated areas of break and short-circuit

Fig. 2 shows that the classical morphological operations caused the closing of the initial open-circuit and breaking of the short-circuit on the original noisy image. If the filtered image is input to PCB inspection system, these defects won't be able to find. Thereby, the disadvantage of classical morphology filters is a lack of sensitivity, i.e. merging of objects with close location apart or breaking of tiny images. In this case, “close location” of the objects should be understood as a distance less than the radius of the structuring element. For example, the minimum symmetrical structuring element is a diamond with the size of a 3 x 3 pixel and if the distance between the object is equal of 1 pixel, there will be a distortion of topology. Using soft morphology operators with additional parameter of the filtration threshold saves the topology of PCB, even if objects on the image are spaced one pixel apart.

Conclusion

In the paper is proposed filtering algorithms based on soft mathematical morphology operators, which characterized by presence of additional parameters such as threshold and the number of iterations. A distinctive feature is the possibility to remove the noise such as islands and pin holes, mousebites and spurs in a single pass without the use of sequential filters. Application of these algorithms can introduce less distortion at work, even if the objects in the image are located at a distance of 1 pixels apart.

Experimental testing of soft morphological filter on example of the PCB artwork image has shown better results than classical morphological filters.

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